Matched Interleavers for Turbo Codes With Short Frames

François Chan
Dept. of Elec. & Comp. Eng.
Royal Military College of Canada
PO Box 17000, Stn. Forces
Kingston, ON., K7K 7B4
email: chan-f@rmc.ca

***Abstract* - The selection of the interleaver has a major influence on the performance of turbo codes with short frames. In this paper, block interleavers are considered. The best block interleaver matched to a given turbo code is obtained by determining the number of columns of the interleaver that will provide the largest effective free distance. It will be shown that at high signal-to-noise ratios a block interleaver with the appropriate number of columns outperforms a random interleaver and even some good interleavers such as the dithered golden interleaver. By combining the search for the code and interleaver, good component codes have been obtained.**

I. INTRODUCTION

Berrou et al. [1], [2] have shown that with iterative decoding, parallel concatenated codes or turbo codes can achieve an error performance very close to the Shannon limit. Iterative decoding of turbo codes can be realized using either the Bahl et al. Maximum A Posteriori (MAP) algorithm [3] or the Soft Output Viterbi Algorithm (SOVA) [4]. Berrou's 16-state code can achieve a bit error rate of 10^{-5} at a signal-to-noise ratio (SNR) of Eb/N0 = 0.7 dB provided a MAP decoder with 18 iterations and a large interleaver of 65536 bits are used [1]. However, the error performance of turbo codes depends on the size of the interleaver and the error probability is larger when a smaller interleaver is used [5].

Furthermore, an "error floor" is observed at a probability of error of 10^{-5} - 10^{-6} approximately depending on the size of the interleaver [5], [6]. Perez et al. [5] have shown that the error floor observed with turbo codes is caused by the relatively low free distance of the code. The error performance can be improved by increasing the interleaver size or the free distance of the code.

Increasing the interleaver size leads to longer delays and larger memory requirements, which may not be desirable for some applications. Furthermore, when the frame is short, such as in mobile radio systems where block sizes are typically under 300 bits, increasing the interleaver size is not possible. Increasing the free distance of the code can be done in two ways, either by using a code with a longer constraint length or by using a better interleaver. A code with a longer constraint length and a larger free distance could compensate for the reduced frame length or interleaver size and improve the error floor. Simulations results show that a code with a larger number of states provides a better error performance than a code with a smaller number of states when the signal-to-noise ratio is high [9]. Hence, for short frames, increasing the memory of the code may be a better alternative than increasing the number of iterations [9].

The second way to increase the free distance is to use a better interleaver. The interleaver plays a key role in the performance of a turbo code system. When an input sequence generates a low weight sequence at the output of the first encoder, the interleaver must permute the input sequence so that a low weight sequence is not generated by the second encoder. For long frames, the choice of which particular interleaver should be used is not too critical and a pseudo-random interleaver can provide a very good performance. However, for short frames, the performance obtained with the pseudo-random interleaver can be improved using different interleavers. Block interleavers will be considered in this paper since they can provide a better error performance than random interleavers for short frames [7].

II. BLOCK INTERLEAVERS

The weight of the information sequence of a recursive systematic code must be larger than 2 in order to yield an output sequence of finite weight [6]. Hence, all error events are caused by information sequences of weight 2 or greater. The minimum weight d_2 of the output sequence for an input sequence of weight 2 is often used to determine the performance of a turbo code [7], [8]. We will compare the minimum weights obtained with block interleavers having from 1 to 192 columns. Therefore, depending on the number of columns, the block may not be a square or a rectangle.

First, a block interleaver with i columns, where i varies from 1 to 192, is considered. An input sequence of weight 2 is then generated. This sequence is interleaved and encoded and the weight of the output sequence is determined. This process is repeated for all possible input sequences of weight 2 and the frequency of all output sequence weights is recorded. The number of columns i is incremented and the whole process is repeated.

The distance spectrum of the 16-state code with generators 23, 35 is determined using all possible block interleavers. For a frame with 192 information bits, an interleaver with 28 colums yields an effective free distance of 20 for a rate 1/3 code. The effective free distance is defined as the minimum weight of code sequences generated by input sequences of weight 2 [8]. By looking at the distance spectrum of all possible interleavers, it has been observed that the effective free distance can vary from 4 to 25 for a rate - 1/3 code with a frame of 192 bits. Hence, it is important to determine the appropriate number of columns for the block interleaver.

Table I presents the distance spectrum of some of the best interleavers. Simulations have been performed to compare the error probability of the 16-state code with different block interleavers and a dithered golden interleaver [13]. This interleaver is known to have excellent spreading properties [13]. The max-log-APP (APosteriori Probability) decoder from the Communications Research Centre (CRC) for the (23, 35) code has been used to compare the performance of different interleavers [11]. It can be seen from Fig. 1 that the golden interleaver is slightly better at medium bit error rates. However, for small bit error rates, the 28-column block interleaver outperforms the golden interleaver. At a BER = 10^{-8}, the block interleaver with 28 columns provides approximately a gain of 1 dB over the golden interleaver. The block interleaver with 28 columns also outperforms a random interleaver.

It can also be seen the 28-column block interleaver is superior to the 51-column interleaver although the latter has a larger d_2. However, d_3, the weight of the output sequence sequence for an input sequence of weight 3 is smaller for the 51-column interleaver.

III. SEARCH FOR GOOD CODES

Benedetto, Garello and Montorsi [8] have presented good component codes for turbo codes. Their codes have been found by using a uniform interleaver.

Better codes have been found by performing a search over all possible generators for each block interleaver. Hence, the code and interleaver design is combined. This is quite similar to the work of Yuan et al. [10] except that they have performed the search for the component codes using a uniform interleaver and then, they have determined an interleaver matched to that code. Here, the code search uses the actual interleaver and not a uniform interleaver.

As an example, Table II compares the distance spectrum of some of the codes found using this technique with the BGM (Benedetto, Garello and Montorsi) codes. It can be seen that BGM codes can yield a large effective free distance if the appropriate block interleaver is used. However, it is possible to achieve a larger distance by using a different code. Fig. 2 compares the performance of the (23, 35) code with the BGM code. It can be seen that the (23, 35) code is slightly better at a small BER.

IV. CONCLUSION

The best block interleavers for turbo codes have been determined by examining the distances obtained with all possible block interleavers. The best interleavers outperform some known interleavers such as the dithered golden interleaver. Good component codes have also been presented. These codes have been found by combining the search for the codes and interleavers.

REFERENCES

[1] C. Berrou, A. Glavieux, and P. Thitimajshima, "Near Shannon limit error-correcting coding and decoding: Turbo-codes", in Proc. ICC 93, Geneva, Switzerland, May 1993, pp. 1064-1070.

[2] C. Berrou and A. Glavieux, "Near optimum error correcting coding and decoding: Turbo-codes", IEEE Trans. Comm., pp. 1261 - 1271, Oct. 1996.

[3] L.R. Bahl, J. Cocke, F. Jelinek, and J. Raviv, "Optimal decoding of linear codes for minimizing symbol error rate", IEEE Trans. Inform. Theory, pp. 284-287, Mar. 1974.

[4] J. Hagenauer, E. Offer, and L. Papke, "Iterative decoding of binary block and convolutional codes", IEEE Trans. Inform. Theory, pp. 429-445, Mar. 1996.

[5] L.C. Perez, J. Seghers and D.J. Costello, "A distance spectrum interpretation of Turbo codes", IEEE Trans. Inform. Theory, pp. 1698-1709, Nov. 1996.

[6] P. Robertson, "Illuminating the structure of parallel concatenated recursive systematic (TURBO) codes", in Proc. GLOBECOM '94, San Francisco, CA, Nov. 1994, vol. 3, pp. 1298-1303.

[7] P. Jung and M. Nasshan, "Dependence of the error performance of turbo-codeson the interleaver structure in short frame transmission systems," Electronics Letters, 17th Feb. 1994, pp. 287-288.

[8] S. Benedetto, R. Garello, and G. Montorsi, " A search for good convolutional codes in the construction of Turbo codes," IEEE Trans. Comm., pp. 1101-1105, Sep. 1998.

[9] F. Chan, "Adaptive Viterbi decoding of turbo codes with short frames," in Proc. IEEE Commun. Theory Mini-Conference, in conjunction with ICC'99, 6-10 June 1999, pp. 47-51.

[10] J. Yuan, B. Vucetic, and W. Feng, "Combined turbo codes and interleaver design," IEEE Trans. Commun., pp. 484-487, April 99.

[11] "Ultra-fast turbo and Viterbi decoders for PCs", CD-ROM, available from the Communications Research Centre (CRC), Ottawa, Canada. See www.crc.ca/fec.

[12] K.J. Larsen, "Short convolutional codes with maximal free distances for rate 1/2, 1/3 and 1/4," IEEE Trans. Inf. Theory, pp. 371-372, May 1973.

[13] S. Crozier, J. Lodge, P.Guinand, and A. Hunt, "Performance of Turbo-codes with relative prime and golden interleaving strategies," in Proc. Sixth Intern. Mobile Satellite Conf. (IMSC `99), Ottawa, Canada, June 1999, pp. 268-275.

TABLE I
DISTANCE SPECTRUM OF SOME OF THE BEST BLOCK INTERLEAVERS

Number of columns	d_2, n_2	d_3, n_3
25	17, 1	24, 1
28	20, 1	23, 4
51	25, 1	15, 3

TABLE II
DISTANCE SPECTRUM OF SOME GOOD COMPONENT CODES

Number of states	G1	G2	N. of columns	d_2, n_2	d_3, n_3
4*	7	5	28	20, 6	12, 1
4	7	6	14	20, 1	13, 1
16	23	35	28	20, 1	23, 4
16*	23	37	28	19, 1	20, 4

* denotes a BGM (Benedetto Guido Montorsi) code.

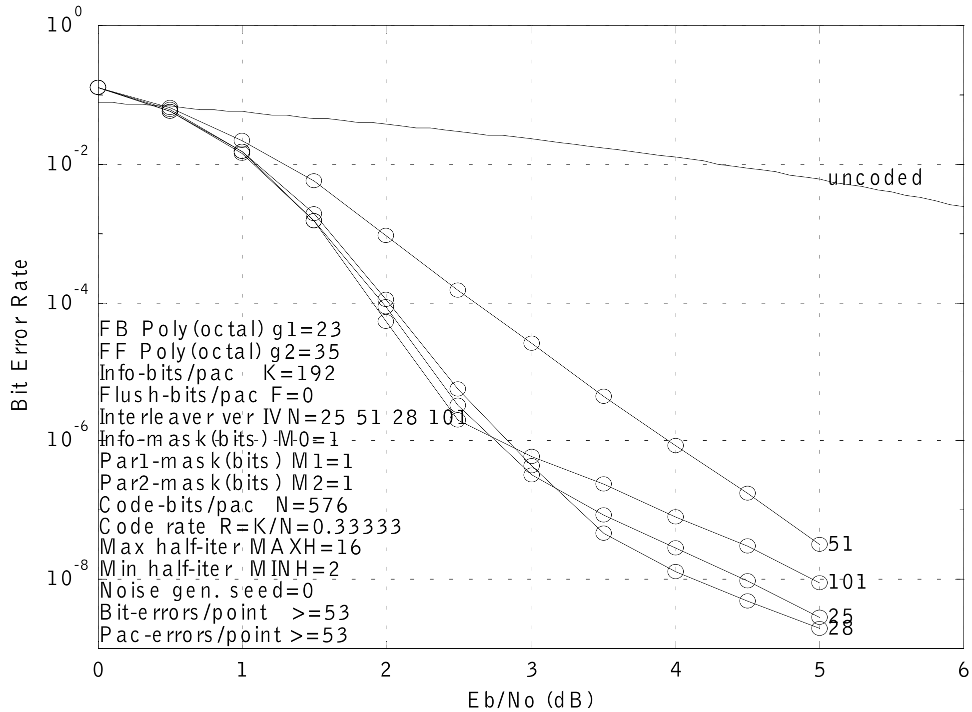

Fig. 1. Comparison of the BER performance for different block interleavers with 51, 25 and 28 columns and a dithered golden interleaver (101).

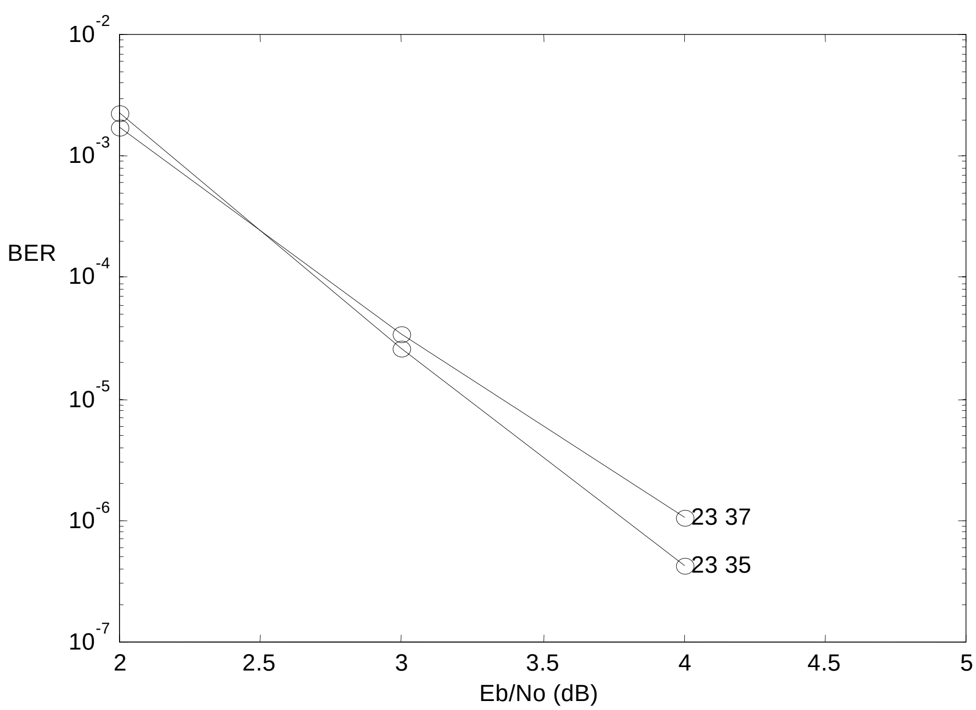

Fig. 2. Comparison of the BER performance for the BGM code and the (23, 35) code – N = 192, block interleaver with 128 columns, 2 iterations.