Stopping Rules for the Bit Error Probability Minimization in Iterative Decoding

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Abstract - In this paper, we propose a simple stopping criterion applied to the iterative decoding algorithm used in concatenated codes with interleaver. Differently from other criteria reported in literature, which attempt to reduce the average number of iterations with little performance degradation, the proposed stopping rules try to minimized the Bit Error Probability (BEP), together with number of iterations.

I. Introduction

In [4], a new coding scheme called "turbo code" showed performance close by 0.5 dB to the Shannon capacity limit, at a bit error probability of 10^{-5}. Successively, the original structure of parallel concatenation of convolutional encoders joined by an interleaver acting on information bits, has been extended to other forms of concatenation [2][1]. The common aspect to the different structures is a suboptimum iterative decoding algorithm [3], whose complexity is linear with the state spaces of the two (or more) constituent convolutional encoders.

Quite often, the number of decoding iterations (N_{it}) is fixed and all frames are decoded with N_{it} iterations. N_{it} is chosen by taking into account both the BEP performance and the decoder speed compared with the data rate.

Stopping rules, i.e., techniques to stop the iterations performed by the decoder based on some suitable criterion, allow to choose the number of iterations to be performed in a frame by frame fashion. Several criteria have been proposed in the literature; some of them are based on soft quantities (like Cross Entropy)[9][5][6], whereas others use hard information [5]. All of them aim at minimizing the average number of iterations while keeping to a minimum the performance degradation. The stopping rules proposed so far work reasonably well when the decoding algorithm shows a uniform behavior, i.e, the BEP decreases with the number of iterations and the signal-to-noise ratio. On the other hand, the iterative decoding algorithm presents sometimes an oscillating behavior, i.e., the number of bit in error per erroneous frame fluctuates with the number of iteration and/or the signal-to-noise ratio. In the latter case, it is important to stop iterating near the minimum number of bit errors. This can be done by choosing a suitable criterion. This paper deals with a comparison between several proposed stopping rules, and the description of a new criterion whose goal is the minimization of the number of iterations together with the minimization of the number of bits in error per erroneous frame. Moreover, some preliminary statistics on the residual bit error distribution at the output of the decoding algorithm will be presented.

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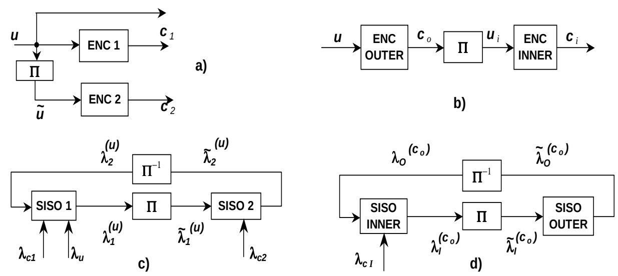

Fig. 1. Structure of parallel (a) and serial (b) encoders and decoders(c) (d).

II. Concatenated codes and iterative decoding

In this section, we present the co-decoding and channel models used in the paper. In Fig. 1 the block diagram of the parallel [4] and serially concatenated codes are shown [2]. In both schemes the sequence $\mathbf{u} = \{u_1, \cdots, u_K\}$ represents the K information bits coming from the source. The first systematic convolutional encoder shown in the parallel concatenation(Fig. 1a)(ENC 1) computes the parity bit sequence $\mathbf{c}_1$, whereas the second convolutional encoder (ENC 2) computes the parity bit sequence $\mathbf{c}_2$, starting from the permuted version of $\mathbf{u}$. The coded sequence is formed by concatenating in some form the three output sequences$(\mathbf{u}, \mathbf{c}_1, \mathbf{c}_2)$. In the serial concatenation a rate k/n code is obtained by cascading an *outer* code (ENC OUTER) with rate k/p (which computes the sequence c_o)and an *inner* code (ENC INNER) with rate p/n (which computes the sequence c_i) through an interleaver of size $N = K \cdot p/k$. In this case, we send to the channel the sequence $\mathbf{c}_i$. We assume BPSK (or 2-PAM) transmission over an AWGN channel characterized by a zero mean random process with power spectral density

$\sigma^2 = N_0/2$. The received sequence $\mathbf{y}$ is transformed by the soft demodulator into the sequence of log-likelihood ratios $\lambda = \log \frac{P(u=1|y)}{P(u=-1|y)}$ to be fed to the soft decoder.

In Fig. 1 the iterative decoder scheme is reported for the parallel(c) and serial(d) concatenations. They are characterized by two soft-input soft-output (SISO) decoders that implement the BCJR algorithm for each of the two constituent encoders [3]. In the figure the λ values are log-likelihood ratios defined in general as

$$\lambda_k^{(a)} = \log \frac{p(a=1|\cdot)}{p(a=0|\cdot)} \tag{1}$$

-Parallel concatenated decoder
In the parallel decoder the received sequence is demultiplexed in three sequences, λ_{c1}, λ_{c2} and λ_u, respectively, pertaining to the two decoders. At i-th iteration, the SISO 1 generates the extrinsic information ($\lambda_1^{(u,i)}$) based on three soft inputs, $\lambda_2^{(u,i-1)}$, λ_{c1} and λ_u, where λ_{c1} to refer at the channel output for the sequences $\mathbf{c}_1$ generate by the encoder 1 and λ_u for the sequence $\mathbf{u}$. In the first iteration, extrinsic information $\lambda_2^{(u,i-1)}$ is zero. The SISO 2 generates the extrinsic information ($\widetilde{\lambda}_2^{(u,i)}$), from two soft input, $\widetilde{\lambda}_1^{(u,i)}$ and λ_{c2}, where λ_2 to refer at the channel output for the sequence $\mathbf{c}_2$ generate by the encoder 2.
-serially concatenated decoder: In the serially decoder the SISO INNER is feed by the log-likelihood ratio (λ_{c_i}),that include the λ_{c_o}, and by the extrinsic information ($\lambda_O^{(c_o,i-1)}$) generate by the SISO OUTER in the previous iteration, and compute the extrinsic information ($\lambda_I^{(c_o,i)}$). The SISO OUTER is feed by the extrinsic information ($\widetilde{\lambda}_I^{(c_o,i)}$) that is the permuted version of the SISO INNER output, and compute the extrinsic information ($\widetilde{\lambda}_O^{(c_o,i)}$). In the first iteration, extrinsic information $\lambda_O^{(c_o,i-1)}$ is zero.

III. Review of existing stopping rules

In this section we briefly describe four stopping criteria already proposed in the literature .
-Cross Entropy (CE) criterion[9]: The CE criterion is based on the following measure of the difference between two probability distributions $P(\mathbf{u})$ and $Q(\mathbf{u})$:

$$T^{(i)} \triangleq \mathrm{E}_p \left\{ \log \frac{P(\mathbf{u})}{Q(\mathbf{u})} \right\} \tag{2}$$

where E_p is the expectation operator over the distribution $P(\mathbf{u})$. Define now the two quantities at the output of the SISO module (the first pair referring to the parallel case, and the second to the serial one):

$$L_1^{(i)} = \lambda_u + \lambda_2^{(u,i-1)} + \lambda_1^{(u,i)}; L_2^{(i)} = \lambda_1^{(u,i)} + \lambda_2^{(u,i)} \tag{3}$$

$$L_1^{(i)} = \lambda_{c_o} + \lambda_O^{(c_o,i-1)} + \lambda_I^{(c_o,i)}; L_2^{(i)} = \lambda_I^{(c_o,i)} + \lambda_O^{(c_o,i)} \tag{4}$$

where the sign of $L_j^{(i)}$ represents the hard decision on the j-th bit, and the magnitude $|L_j^{(i)}|$ is related to the reliability of the decision. The difference between the soft outputs is equal to

$$\Delta L \triangleq L_1^{(i)} - L_2^{(i)} \tag{5}$$

and the CE between the two distributions $P(\mathbf{u}) = P[L_1^i]$ and $Q(\mathbf{u}) = P[L_2^i]$ can be approximated as

$$T^i \approx \sum_{k=1}^{(K)} \frac{(\Delta L)^{(2)}(u_k)}{e^{|L_2^{(i)}(u_k)|}} \tag{6}$$

When the difference between the values of $T^{(i)}$ at two successive iterations lies below a suitable threshold S_t we stop the iterative process.
-Sign-Change-Ratio (SCR) criterion[5]: This criterion is simpler and easier to implement than the CE. It computes the number of sign changes $N_{sc}(i)$ in the extrinsic information ($\lambda^{(u,i)}$)(or $\lambda^{(c_o,i)}$ in the serial case) between two successive iterations, and stops the decoding process when $N_{sc}(i) \leqslant S_t$. The threshold S_t is made equal to qK, where q is a constant usually chosen to lie in the range $0.005 \leqslant q \leqslant 0.03$, and K is the frame size. The choice of the threshold value is a trade-off between the BEP (that decreases with a decreasing of the threshold), and average number of iterations (that increases with a decreasing of the threshold).
-Mean-Estimate (ME) criterion[8]: This criterion monitors the quantity

$$M_{|\lambda_m^{(u,i)}|} = \frac{1}{K} \sum_{k=1}^{(K)} |\lambda_m^{(u,i)}|,$$

i.e., which is related to the reliability of decoding the overall frame, and stops iterating when $M_{|\lambda_m^{(u,i)}|} > S_t$. Increasing the value of the threshold S_t makes the average number of iterations increase, and the BEP decrease.

In all cases, a maximum number of iterations is defined, after which the decoding process is stopped even though the stopping rules have not been satisfied.

IV. A new stopping rule

In this section, we discuss the previous stopping criteria and introduce the new stopping rule.

As already mentioned, the thresholds of the previously described stopping rules comes from a trade-off between average number of iterations and BEP.

As ane example, we show in Fig. 2 the number of errors as a function of the number of iterations in two typical frames for $E_b/N_0 = 1.4$ dB. In the first case (1) the number of bit errors decreases monotonously with the number of iterations, and at the eleventh iteration it becomes zero. Adopting in this case the SCR criterion with a threshold higher than 12 would stop the decoder at tenth iteration, and therefore with 21 errors. The same result would be obtained using the ME criterion with a threshold less than 60. To guarantee the correction of correctable frames with a monotonous decreasing of the bit errors per frame versus the number of iterations one should use a threshold zero for the SCR rule and a sufficiently high value (from our simulations 60 is a good number) for the ME criterion.

The second case, shown as (2) in the figure, exhibits an oscillating behavior, i.e., the number of bit errors fluctuates versus the number of iterations. In this case, a threshold 0 and 60 for the two criteria SCR and ME does not stop the iterative process, and then the decoding algorithm performs its maximum number of iterations N_{it}. The number of bit errors is then the one obtained at the end of the process, which may be very high. The stopping rule we propose permits avoiding these situations, and thus it yields both a decreasing of the average number of iterations and of the BEP.

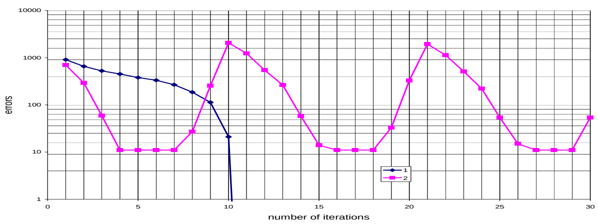

Fig. 2. Number of bit errors for two typical frames versus the number of iterations: (1) refers to a monotonously decreasing behavior, while (2) shows an oscillating behavior.

The new stopping rule is based on the sign of the extrinsic information: the number of sign changes in the extrinsic information between two consecutive iterations, $N_{sc}(i)$, plotted versus the number of iterations, agrees with the curve of bit errors versus number of iterations. Then, we can stop the decoding process when N_{sc}, after decreasing under a threshold (S_t), increases again above a second (hysteresis) larger threshold S_h. This rule (that we call HT) permits to get rid of the oscillating behavior, by stopping the number of iterations around a minimum of the curve, thus improving the BEP. On the other hand, for frames not exhibiting an oscillating behavior, this criterion would not stop the number of iterations. So, overall, the HT criterion alone would decrease very slightly the average number of iterations.

To accomplish both a reduction of the number of iterations and of the BEP, we can associate the HT rule with the SCR using a threshold 0 (we call this HT-SCR).

V. Simulation results

To compare the different stopping criteria, we have simulated a rate 1/2 serially concatenated code made up of two 4-state constituent encoders. Both are obtained (the outer with rate 2/3, and the inner 3/4) by puncturing a mother recursive systematic encoder with rate 1/2 and parity-check polynomials (in octal form) (1,5/7). The frame length is 8,000 information bits.

In Fig. 3 we show the BEP and FEP curves for the cases of the standard algorithm with a fixed number of 10 iterations, and the HT rule working with $N_{it} = 10$. We notice that the FEP are exactly the same, whereas

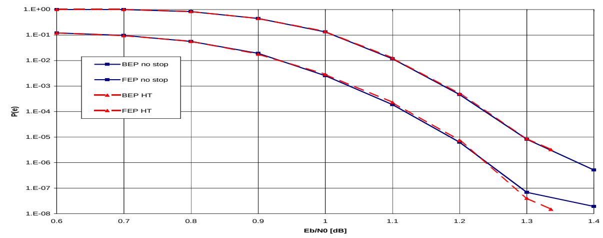

Fig. 3. Bit an Frame error probability performance for the standard decoding algorithm with a fixed number of iterations (10) and for the HT stopping rule.

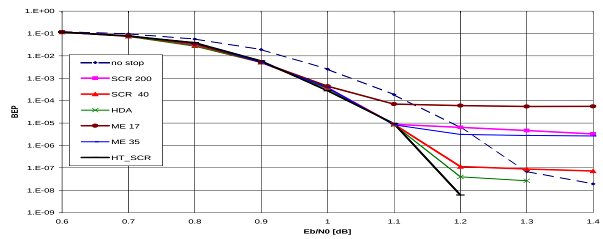

Fig. 4. Bit error probability performance for several stopping criteria using $N_{it} = 100$ and for the standard decoding algorithm with a fixed number of iterations (10).

the BEP shows an error floor for the standard algorithm, which is not present in the HT case. This different behavior is due to the reduced number of bit errors per erroneous frame in case of oscillating behavior.

In Fig. 4 and Fig. 5 we show the BEP and FEP performance for several stopping rules, i.e., SCR, ME, and HT-SCR with $N_{it} = 100$. We have also included for comparison the performance if the decoder working without stopping rules and using a fixed number of 10 iterations. The corresponding average number of iterations is shown in Fig. 6. Some comments are appropriate. First of all,

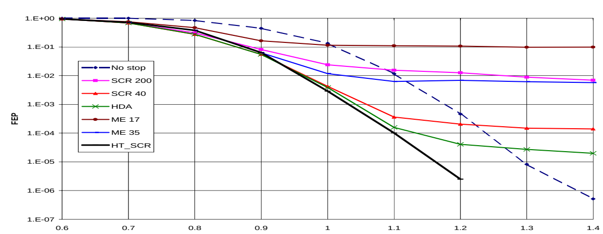

Fig. 5. Frame error probability performance for several stopping criteria using $N_{it} = 100$ and for the standard decoding algorithm with a fixed number of iterations (10).

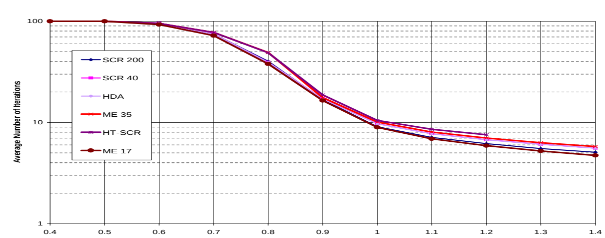

Fig. 6. Average number of iterations for several stopping criteria versus signal-to-noise ratio.

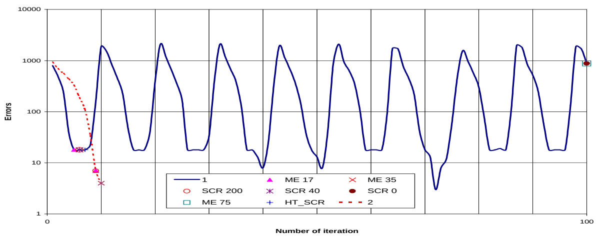

Fig. 7. Stopping points (expressed as number of bit errors) of three different stopping rules for two typical frames: (2) refers to a monotonously decreasing behavior, while (1) shows an oscillating behavior.

from Fig. 6 we see that all criteria permit to reduce the average number of iterations below 10 for signal-to-noise ratios above 1 dB, even though $N_{it} = 100$.

Turning now to the curves of Fig. 4 and Fig. 5, we notice, for the SCR criterion, corresponding to two thresholds, $S_t = 280$ and 40), the effect of the threshold in the error floor region, i.e., where the BEP and FEP change their slope. In fact, when the threshold is set to 40, the error-floor region starts at 1.1 dB, while for $S_t = 280$ it starts at 1.2 dB.

The ME criterion [8] should use a different threshold for each signal-to-noise ratio. To simplify the simulations, we have used a fixed threshold optimized in our simulation in correspondence of BEP around 10^{-5} ($S_t = 17$) and 10^{-7} ($S_t = 35$). From the curves, we notice that for $S_t = 17$ the error floor region begins at 1.0 dB, while for $S_t = 35$ it starts at 1.1 dB.

The new HT-SCR composite criterion shows the best performance for all E_b/N_0. On the other hand, for $E_b/N_0 \geq$ 1.3 dB, the BEP and FEP are too low $(10^{-9}, 10^{-6})$ to be reliably simulated.

All criteria, above the error floor region, yield significant gains with respect to the standard algorithm with a fixed number of iterations.

From previous results, one would conclude that the threshold should be set to 0 for the SCR and to a very large value for the ME criterion, in order to reduce the error floor. On the other hand, this would make those criteria sensitive to the frames exhibiting an oscillating behavior. Also the SCR would exhibit the same behavior, at BEP (and FEP), however, below our present simulation capabilities. For this reason, we have isolated two sample frames, and verified the various stopping rules on them, as shown in Fig. 7. In the first case (1), referring to the oscillating frame, the SCR (ME) stops the iterative process at fifth iteration for sufficiently high (low) thresholds, whereas they do not stop the decoder for zero (very high) thresholds. Thus, if we want to stop the iterations for a minimum number of bit errors in oscillating frames, we incur in high error floors. In the second case (2), the SCR and ME criteria stop the iterative process where the number of error is not zero, for high (low) thresholds. Our criterion, on the other hand, stops the decoder in both cases at the lowest number of bit errors (18 bit errors for the first case, and 0 for the second one).

TABLE I

PROBABILITY THAT THE FRAME IS STOPPED WITH A GIVEN RESIDUAL BIT ERRORS

bit err.	SCR 40	SCR 200	ME 35	ME 17	HT-SCR
1-5	$2E-4$	$1E-2$	$6E-3$	$8E-2$	0
6-10	$6E-5$	$2E-3$	$2E-4$	$1E-2$	0
11-20	$1E-5$	$6E-4$	$3E-5$	$5E-3$	$1E-5$
21-30	0	$1E-5$	$1E-5$	$5E-4$	0
700-900	$1E-5$	$1E-5$	$5E-5$	$5E-5$	$1E-5$

VI. Bit error statistics

In Table I we show the probability that the frame is stopped with a given residual number of errors for several stopping rules at $E_b/N_0 = 1.1$ dB. The SCR criterion presents a high probability that the frame is stopped with a low number of residual error; this probability decreases if the threshold is reduced. The ME criterion presents the same behavior. The HT-SCR instead does not present frame with a low number of errors, therefore is not necessary to introduce an outer encoder and to correct it the residual errors.

The number of bit errors shown in the last row of Table I is high because there are frames that cannot be corrected by iterative decoding, independently from the number of iterations.

VII. Conclusion

In this paper, we have proposed a simple stopping criterion applied to the iterative decoding algorithm used in concatenates codes with interleaver. Compared to the performance of other criteria reported in the literature, it minimizes the bit error probability and yield about the same average number of iterations, with a slight increase of computational complexity.

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